Equity and Fairness in Quantitative Employee Rating Systems

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Introduction. Quantitative employee rating systems were sharply criticized by the late W. Edwards Deming and his followers [1,2]. Research addressing these systems continues and new versions of quantitative rating systems are being studied empirically and simulated using computers [3,4,5].

In a quantitative employee rating system a single numerical score, or a finite set of numerical values, is assigned to the employee at the end of a defined period of performance, and this single score or set of numerical scores determines the employee's salary for the next period of employment.

Deming criticized such rating systems, calling them a "deadly disease of organizations", because he believed that performance scores could not be developed that reflected only the employee's contribution to the work effort. Rather, Deming believed that an unknown portion of an employee's score always reflected random variation in the corporate process itself. Thus, rewarding (or penalizing) the employee on the basis of a performance score at least partially rewards or penalizes her for random factors beyond the employee's control. Deming recommended use of a qualitative assessment system that communicates corporate expectations and hopes to the employee, reinforces employee behaviors that are desired and counsels against behaviors and work processes deemed non-productive. Deming seems to suggest that an individual's salary should reflect more the group contribution to the organization than the specific performance or contribution of the individual [1].

Deming's critique of the use of quantitative performance scores in the assignment of salary prompts difficult questions of fairness to individuals and important issues about the kind of corporate environment differing employee assessment and salary schemes engender.

If the use of a performance score randomly over or under compensates an employee due to failure to recognize and understand process variability, can the supervisor and employee expect these random effects to balance out over the course of time? Can the employee and supervisor expect the system to be fair and accurate when considered over time? What can a manager do to insure that system variability falls equally on all employees so that there might be salary equity over time? Has it been demonstrated that qualitative systems of employee assessment are less susceptible to random fluctuations beyond the employee's control than are quantitative assessments? Does rewarding an individual primarily on the basis of group performance foster the kind of initiative and creativity desired in the workplace?

In this article we accept that quantitative performance or contribution assessment and salary assignment are used and probably will continue to be used in various organizations. Our goal is to point out equity and fairness concerns that arise in practical implementation. This analysis uses accounting mathematics and assessment techniques.

A simple quantitative employee assessment and salary system. A simple and direct quantitative employee assessment and salary assignment system would aim at exact and strict financial impartial fairness. For example, suppose an employee received $33,000 during the period of performance (usually one year) and his performance assessment score for that period indicates a performance or contribution value of $35,000. Under a simple strict system, the employee would receive an additional $2,000 for his or her period of performance. The payment of $2,000 compensates for the employee productivity that was not previously paid for. Management may choose to assign a new salary of $35,000 for the next performance assessment period on the assumption that immediately prior performance is a predictor of subsequent performance.

Similarly, if the employee received $33,000 for the assessment period but an assessment score that indicates a performance value of only $31,000, the employee would return $2,000 to the employer and enter the next assignment period with a salary of $31,000.

This very direct and simple performance assessment and salary scheme encounters two difficulties. First, a salary system which requires an employee to return already received salary, would impose hardship on those employees who fully utilize all of current income. One could imagine a system where employees have a usefully precise idea of their quantitative assessment throughout the course of a performance period, so that they could set aside money for repayment if necessary. Such a system has never been used in practice to our knowledge.

Second, employing organizations may not have the available cash to fully cover the salary value of employees who have received assessment scores requiring payments. This problem can be particularly serious if there is a tendency for managers to inflate the performance or contribution scores of their employees. Also, a performance assessment score or an employee contribution score will rarely, if ever, be a measure of the true monetary value of production. There will be several sources of error, many of which have been discussed by Deming. Does it make sense to withdraw salary from an employee on the basis of an imperfect score? Does it make sense to directly reward an individual on the basis of a performance or contribution score that contains random and perhaps systematic error?

In summarizing terms, the simplest and most direct accounting and payment method using a quantitative employee rating and aiming at strict, impartial fairness encounters the same problems from both the employee and the employer side, namely, the problems of funds limitations and score error. As shown in the next section of this article, attempts to deal with corporate funds limitations for salary can lead to unexpected (and undesired) features in a pay plan.

A cost constrained salary assignment system. One approach to quantitative employee assessment and salary assignment, that constrains employer outlays, is described below using accounting mathematics. The authors are concerned that the use of equations and mathematical symbols will serve as a barrier to the reader who might be interested in personnel systems and policy but is not trained in mathematics. We believe that the use of mathematics is necessary for a complete understanding of the problem we have uncovered. However, we try to make this article accessible to the non-mathematical reader by trying to provide ample English language non-mathematical explanations.

In an approach to quantitative employee assessment and salary assignment that constrains corporate outlays, an employee's new salary, $S_{i+1,j}$, is determined as a function of his or her prior salary, $S_{i,j}$, and performance score $x_{i,j}$. In this notation the subscripts *"j"* refers to the employee, and the subscripts *"i"* and *"i+1"* refer to the first and following performance period respectively. In mathematical notation we write

$$S_{i+1,j} = f(S_{i,j}, x_{i,j}) \qquad \text{(Equation \#1)}$$

This equation simply says that an employee's subsequent salary shall be a function of her prior salary and her prior assessment score. The symbol $f(S_{i,j}, x_{i,j})$ can be called the salary assignment function. This function will generally have the property that it increases as the performance score $x_{i,j}$ increases, obviously reflecting the notion that increased performance or contribution merits increased pay.

A given year's corporate salary outlay will always be a multiple of the prior year's salary costs. In the case where the employee cohort stays the same size from one assessment period to the next, one therefore has the equation.

$$\sum_{j=1}^{M} S_{i+1,j} = \mu \sum_{j=1}^{M} S_{i,j} \qquad \text{(Equation \#2)}$$

The summation notation indicated by the symbol $\sum_{j=1}^{M}$ refers to a summation of salaries over the entire worker cohort involved in the analysis. The symbol "μ" is the salary line multiple which is usually a number greater than one due to cost of living raises and productivity increases in the worker cohort.

Now, for purposes of illustration, a specific salary assignment function $f(S_{i,j}, x_{i,j})$ is chosen. Consider that the performance score directly and linearly reflects deserved salary symbolized $S^{d}_{i,j}$. That is, assume

$$S^{d}_{i,j} = a_i + b_i x_{i,j} \qquad \text{(Equation \#3)}$$

A simple salary assignment function can be defined as including a cost of living raise plus a proportion of the discrepancy between the salary $S_{i,j}$ actually given the employee and the salary $S^{d}_{i,j}$ actually earned or deserved by the employee. Mathematically this translates as

$$S_{i+1,j} = f(S_{i,j}, x_{i,j}) = (1+\lambda)S_{i,j} + \alpha(S^{d}_{i,j} - S_{i,j}) \qquad \text{(Equation \#4)}$$

In this last equation (equation #4), $S^d_{i,j}$ is given by equation #3. The variable λ is the cost of living raise and α is the percentage salary increase provided or deleted reflecting the employee's performance assessment or contribution score. This coefficient, the α coefficient, is critical. It defines the degree to which the employee's performance or contribution is reflected into her salary during the next performance period. We call this coefficient the "contribution coefficient". In a proper pay plan, this contribution coefficient, α, will not be a function of group properties over which the employee has no control.

Using equation #3 in equation #4 leads to

$$S_{i+1,j} = (1+\lambda)S_{i,j} + \alpha(a_i + b_i x_{i,j} - S_{i,j}) \qquad \text{(Equation \#5)}$$

Direct substitution of equation #5 into equation #2 leads to the result that the contribution coefficient α, the fractional salary increase (or decrease) provided an employee reflecting an employee's score, becomes a complex function of the <u>entire</u> cohort's total salary cost and performance scores. Specifically, simple algebraic work yields.

$$\alpha = \frac{(\mu - \lambda - 1)\left(\sum_{j=1}^{M} S_{i,j}\right)}{a_i M + b_i \left(\sum_{j=1}^{M} x_{i,j}\right) - \left(\sum_{j=1}^{M} S_{i,j}\right)} \qquad \text{(Equation \#6)}$$

This last equation means that the value of an employee's performance or compensation score, measured by the coefficient α, is a complex function of the cost of the total salaries paid in his or her work group and is also dependent on the performance scores $x_{i,j}$ of his or her colleagues.

This last equation, equation #6, violates what we understand as the spirit, if not the letter, of equal opportunity law and equal-pay-for-equal work law. As we understand these laws, an employee's pay should <u>not</u> depend on factors beyond his control such as race, gender, or any other property of group membership beyond her control. It seems to us that an employee's performance or contribution score should be worth a dollar amount that depends only on the value of the employee's performance, and, should be independent of properties of the group to which she belongs and which she cannot control.

Equation #6 indicates a serious problem in pay plans operated in a setting wherein there are corporate constraints on salary outlays. We have been unable to find a salary assignment function $f(S_{i,j}, x_{i,j})$ that does not have this property of rendering employee compensation a function of the salaries earned by his colleagues and the performance scores received by his colleagues. Even highly nonlinear functions seem to have this property.

Equation #4 is quite good as it stands when the coefficient α is a constant. But, since α is not a constant but dependent upon group properties over which the employee has no control, the cost constrained quantitative employee rating system seems indefensible.

Further study of equation #6, which we call the "alpha equation", can improve understanding of quantitative rating and salary assignment systems. Dividing the numerator and denominator of the alpha equation by M, the number of employees in the pay pool, results in the equation

$$\alpha = \frac{(\mu - \lambda - 1)\overline{S}_i}{(a_i + b_i\overline{x}_i) - \overline{S}_i} \qquad \text{(Equation \#7)}$$

In this equation the term $\overline{S}_i$ is the average or mean salary paid to the group and the term $\overline{x}_i$ is the average score or mean contribution score in the group. Let us examine this function to see the severity of the problem of inappropriate group influence on employee pay.

If, during the pay period, the group's average earned salary is exactly what they were paid on average, one has

$$a_i + b_i\overline{x}_i = \overline{S}_i \qquad \text{(Equation \#8)}$$

Note that, in this situation, the coefficient α becomes indeterminately large! This is obviously a pathological case. Referring back to equation #4, it is clear that the largest value of α that makes sense is the number +1. When α is the number +1, an employee is simply given the difference between what he has actually earned and what he has been paid.

If during the pay period, the group's average earned salary is less than what they were paid on average, the coefficient α becomes a negative number. This is also obviously also a severely pathological case. For if α is a negative number, one has the paradoxical result that an employee that has actually earned more than he has been paid will be penalized, and the employee that has actually earned less then he has been paid will be rewarded! One can readily grasp this last statement by reference to equation #4 and by considering the coefficient α, the contribution coefficient, to take a negative value therein.

Thus, this cost constrained quantitative employee rating system can only work without gross paradox in that case wherein the group on average improves its performance over what it has been paid. That is, the system can only work without gross paradox when

$$a_i + b_i \bar{x} > \bar{S} \quad \text{and} \quad 0 \le \alpha \le 1 \qquad \text{(Equation \#9)}$$

Therefore, we turn our attention to a study of the conditions under which the conditions listed as equation #9 occur.

References #4 and #5 concern simulations of a system that is roughly similar to the one that we have studied here. In that system $a_i = \$14,000$ (approximately), $b_i = \$16,000$ (approximately) and employee contribution scores could range from the number 1 through 5 (thus $1 \le \bar{x}_i \le 5$). In figure #1 we have plotted α as a function of $\bar{x}_i$ and $\bar{S}_i$. We have assumed the organization will increase its pay line by 4.7552% overall, including a 2.3% cost of living raise and 2.4% applied to salary increases associated with productivity or contribution improvement ($\mu = 1.023 * 1.024 = 1.047552$ and $\lambda = 0.023$). Figure #1 is a plot of the coefficient α as a function of average group score and average group salary. This surface plot indicates that α is a strong function of average group score and average group salary, two group properties over which the employee has no control.

Figure #1 is a clear demonstration that cost constrained quantitative employee rating systems can lead to the situation wherein the employees' compensation is not only determined by his or her contribution, but is also dependent upon group properties not under his control. A closer look at Figure #1 reveals some further interesting properties. These are illustrated in Figures #2 and #3.

In Figure #2 we have plotted the compensation coefficient α as a function of average group performance or contribution score for three average group salaries ($50,000, $60,000, and $70,000). If one holds constant the average group performance or contribution score at the number 3.8 (for example), one sees clearly that the compensation coefficient α increases dramatically with group salary. Thus, the "wealthier" the group you are in the greater your rewards despite the same performance as a colleague in a "poorer" group.

In Figure #3 we have plotted the compensation coefficient α as a function of average group salary for three performance or contribution scores (3.0, 3.5, and 4.0). If one fixes the average group salary at the number $60,000 (for example), one sees clearly that the compensation coefficient α increases dramatically as group performance falls. Thus, the "less contributing" the group you are in the greater your rewards may be despite the same performance as a colleague in a "highly contributing" group.

The critical reader will ask whether the phenomena discussed here have ever been observed in actual pay systems. We have a small data set from the system simulated in references #4 and #5, but as actually operated in 1997. The data are shown in the table below.

Group name	Ave group salary	Ave group score	Alpha coefficient
RL/OC	$60,305	2.95	.634
AL/HR	$62,345	3.22	.345
WL/AA	$63,349	3.29	.347
AL/CF	$65,028	3.33	.395

In this small data set both average group salary and average group score are changing simultaneously. The small amount of data available and the fact that the system described in this report is an approximation of the real system used, makes a direct numerical comparison of this data with the predictions of Figure #1 essentially impossible. Please note however, that the values for the coefficient α in the table are similar to those estimated in Figures #1, #2. and #3 for the values of average group salary and average group score involved. Also, the significant variability in α is expected from the steep shape of Figure #1. **However, the most striking fact about the data in the above table is the very change in the coefficient α across the four groups despite the fact that the exact same cost constrained pay plan is used in each group!** This is unambiguous evidence that use of a constrained pay plan system results in differing compensation for the same contribution score, and this is the basic claim of the simple accounting analysis presented here.

Conclusion. Deming has illustrated statistical concerns with quantitative employee rating systems. These concerns are a continuing subject for research. In this report, financial accounting features of quantitative employee rating systems have been studied. We have shown that when the employer constrains his salary expenditures, an individual employee's salary may not simply reflect his or her performance score. Rather, individual salaries can also reflect the group's average salary and average performance score. Both employers and employees may find these features of constrained systems undesirable if not against the spirit and letter of equal opportunity law and equal-pay-for-equal work law.

The theoretical work shown here parallels findings produced by detailed simulation work already published [4,5]. The detailed simulations of an actual system indicated that, over a six-year period, 30% of an employee's salary increment could be attributed to group membership independent of employee performance [5]. Specifically, the correlation coefficient between individual salary and group salary was greater than 0.600. The authors conclude that: "This means that there is, indeed, a positive relationship between the 'wealth' of the pay pool to which one is assigned and one's opportunity for advancement."

We were led to perform this accounting mathematics study of quantitative pay plans because of the warnings written and lectured by Dr Deming. Further, we were concerned about these pay plan systems because they remind us of voting systems. We are reminded of voting systems since within a particular pay group there may be multiple managers who are rating individual employees. It has been long known that systems of social choice that use voting procedures result in paradoxes. These paradoxes appear as the selection of individuals who are in some sense less favored by the group and as the manipulation of an election outcome by voting against one's interest. For example, anomalies resulting from the Electoral College in the United States have been known to occur [6]. Noting the unexpected financial behavior of the constrained quantitative salary programs indicated in this article and in references #4 and #5 suggests to us that these systems need much further study. These systems need a thorough review from the point of view of social choice theory [7,8] as well as further accounting assessments addressing issues of equal treatment.

Fairness, equal treatment, equal pay for equal work, compensation that is independent of group factors like race and gender that the employee cannot control and which do not bear on job performance, are all treasured principles of American democracy. Why should an employee get higher raises just because she is in a group that has a higher average salary? Do we want to give an employee who has performed slightly above average a higher raise simply because he is in a group that has performed poorly on average? These seem to be consequences of using a cost-constrained quantitative rating system. These consequences may not be easily spotted by simple review of data from actual pay plan experience because the conceptual schemes for analysis are lacking. We hope that we have clearly demonstrated that mathematical methods and computer science are excellent tools to aid the personnel professional in the assessment of pay plan systems. Finally, we believe, based on our study of cost constrained quantitative pay plans, that, perhaps, Dr Deming was right when he opined that a fair quantitative rating system is impossible.

THE PERFORMANCE OR CONTRIBUTION COEFFICIENT α AS A FUNCTION OF AVERAGE EMPLOYEE SALARY AND AVERAGE EMPLOYEE PERFORMANCE OR CONTRIBUTION SCORE.

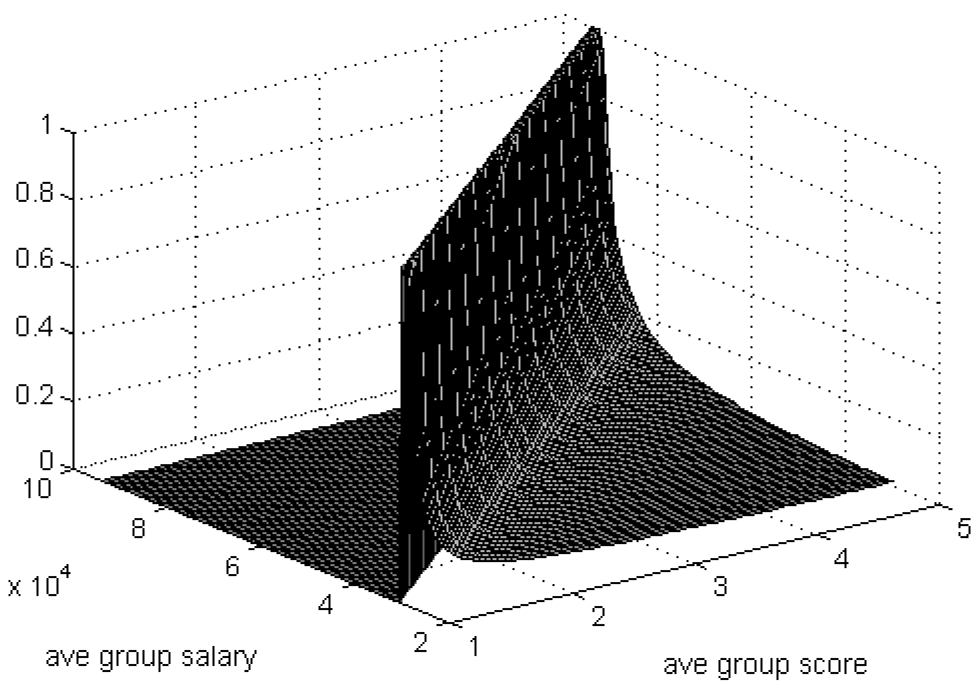

Figure #1. Plotted here is the coefficient α as a function of average group score and average group salary. Note, that when the coefficient does not take the value zero or one, it depends on average group score and average group salary in a striking manner indicating an unacceptable dependence on group properties not within the control of the employee.

THE PERFORMANCE OR CONTRIBUTION COEFFICIENT α AS A FUNCTION OF AVERAGE EMPLOYEE PERFORMANCE OR CONTRIBUTION SCORE.

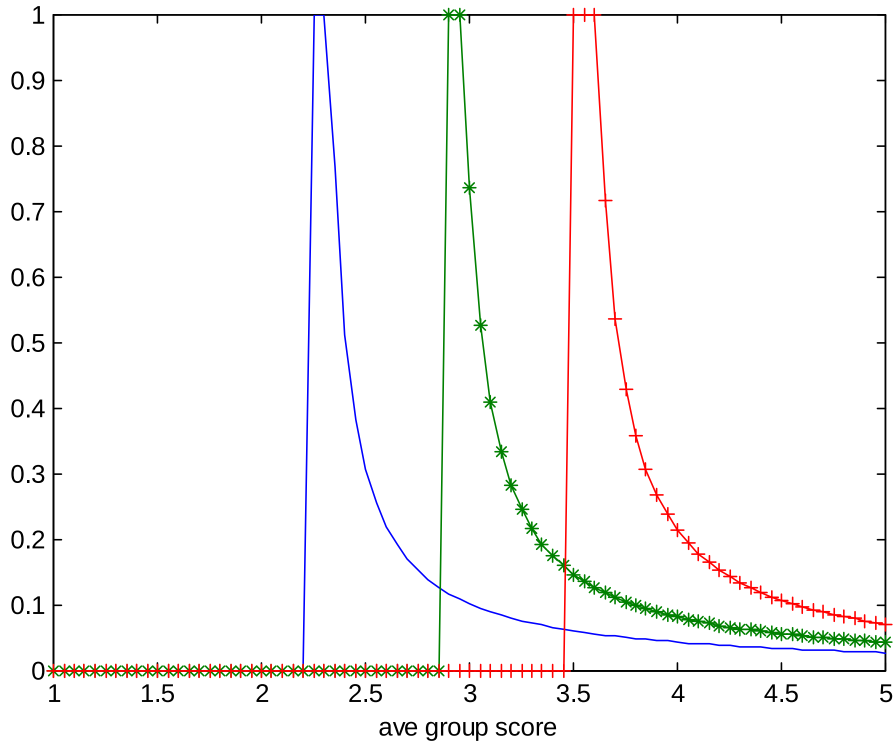

Figure #2. Here we plot the performance or contribution coefficient α as a function of average group performance or contribution score for three different values of average group salary. The solid curve shows the contribution coefficient as a function of average group score with an average group salary of $50000. The central curve, with the solid and asterisked line, shows the contribution coefficient as a function of average group score with an average group salary of $60000. The rightmost curve, with the solid and plus signed line, shows the contribution coefficient as a function of average group score with an average group salary of $70000. Notice that, for a fixed average group score, the employee in the group with the higher average salary receives a higher contribution coefficient (and this translates as increased pay).

THE PERFORMANCE OR CONTRIBUTION COEFFICIENT α AS A FUNCTION OF AVERAGE EMPLOYEE SALARY.

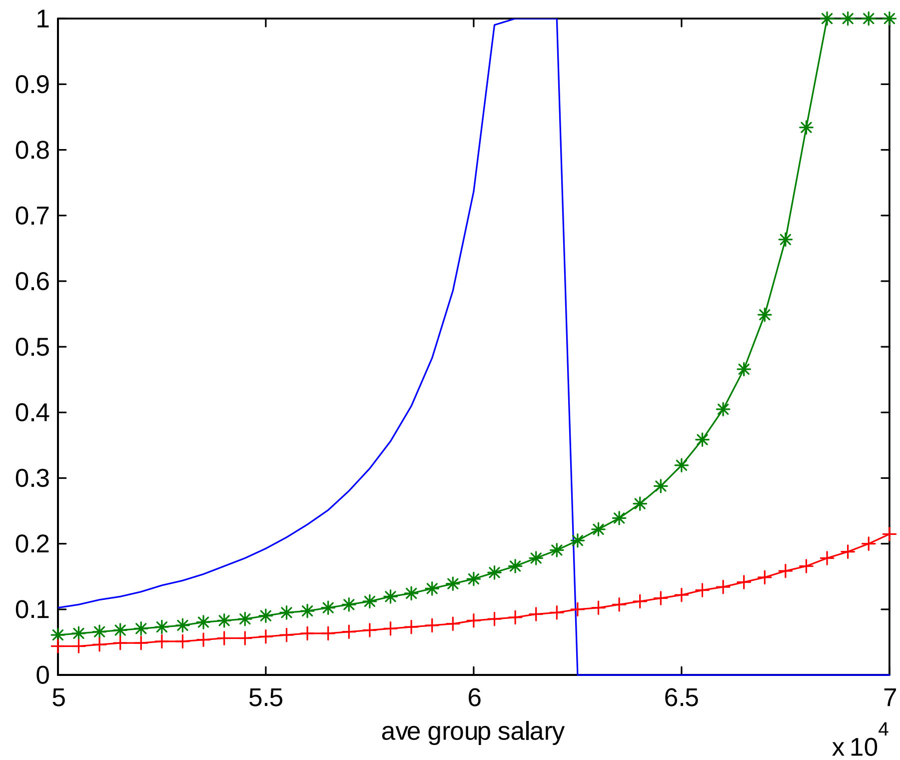

Figure #3. Here we plot the performance or contribution coefficient α as a function of average group salary for three different values of average group performance or contribution score. The solid curve shows the contribution coefficient as a function of average group salary with an average group performance or contribution score of 3.0. The central curve, with the solid and asterisked line, shows the contribution coefficient as a function of average group salary with an average group score of 3.5. The rightmost curve, with the solid and plus signed line, shows the contribution coefficient as a function of average group salary with an average group score of 4.0. Notice that, for a fixed average group salary, the employee in the group with the lowest average performance or contribution score receives the higher contribution coefficient (and this translates as increased pay). This is certainly a paradoxical and concerning result!

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